Math 453 – Combinatorics

Course Description from Bulletin: Permutations and combinatorics; pigeonhole principle; inclusion-exclusion principle; recurrence equations and generating functions; enumeration under group action. (3-0-3)

Enrollment: Elective for AM and other majors.


Other required material: None

Prerequisites: MATH 230 Introduction to Discrete Mathematics, or consent of the instructor

Objectives:
1. Students will be able to generate conjectures from examples and formulate precise conjectures.
2. Students will be able to recognize and write valid proofs. Proof techniques include bijective/combinatorial proofs, induction, and the pigeonhole principle.
3. Students will be able to discuss mathematics, including: presenting solutions at the board, generating examples for illustration as appropriate, seeking and finding holes in proposed proofs.
4. Students will be familiar with common examples including: subsets of a set, functions, onto functions, one-to-one functions, basic graph substructures, Lattice Paths and Catalan Numbers, the Binomial Theorem and Pascal's Triangle, basic Ramsey Numbers, Fibonacci Numbers, multisets and compositions of integers, derangements, colorings of n-gons up to symmetry.
5. Students will be able to count (with proofs) standard and unfamiliar examples using the techniques listed below. Also, be familiar with (proof of) why each technique is valid.
   a. sum rule, product rule, quotient rule
   b. distributions from “the 20-fold way” including permutations, combinations, multisets, compositions of integers, Stirling numbers of the second kind, and partitions of integers
   c. recurrence relations and ordinary generating functions
   d. Principle of Inclusion-Exclusion
   e. groups acting on sets (“Burnside's Lemma” and “Polya Enumeration”)

Lectures: 3 50 minute classes per week, or 2 75 minute classes per week.

Course Outline: Hours
1. Basic Counting Principles and Examples 6-9
2. Applications of Induction and Recursion in Combinatorics 4-10
3. Distribution Problems 6-9
4. Generating Functions 6-9
5. The Principle of Inclusion-Exclusion 3-5
6. Groups Acting on Sets 4-10

Assessment: Problem Solutions 10-40%
           Quizzes 0-10%
           Midterm Exams 20-50%
           Final Exam 30-40%

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