Problem 1. Augment the stack data structure to do in constant time, besides push/insert and pop/remove, findmin. Describe the data structure, and present pseudocode for push/insert, pop/remove, and findmin. Argue that each operation takes constant time. Argue that findmin correctly returns the element with minimum key.

Problem 2. Describe an efficient greedy algorithm for making change for a specified value using a minimum number of coins, assuming there are four denominations of coins (called dimes, nickels, quarters, and pennies), with values 10, 5, 25, and 1, respectively.

The input of your algorithm is an integer number $N$ representing the total number of cents. The output specifies for each denomination how many coins are to be used. Here, “efficient” means $O(N)$.

1. Present pseudocode, running time analysis, proof of correctness.

2. Find four denominations of coins, one of which is 1c, and one value of $N$ for which the greedy algorithm fails to return the minimum number of coins. Show the greedy and optimum solutions.

3. Present a dynamic program for any set of $k$ denominations. Strive for running time of $O(kN)$, but make sure that the running time is polynomial in $k$ and $N$. Present the pseudocode, discuss correctness, and analyze the running time.
Problem 3. A multiple source-sink network is a tuple $G = (V, E, c, S, T)$, where $V$ is a set of vertices, $E$ is a set of directed edges (parallel edges are allowed), $S \subset V$ is the set of sources, and $T \subset V$ is the set of sinks, $c$ is a capacity function: $c : E \rightarrow Z_+$. Also, $S \cap T = \emptyset$. That is, sources are distinct from sinks.

A function $f : E \rightarrow R_+$ is called a flow if the following three conditions are satisfied:

1. conservation of flow at interior vertices: for all vertices $u$ not in $S \cup T$,
   $$\sum_{e \in \delta^-(u)} f(e) = \sum_{e \in \delta^+(u)} f(e);$$

2. capacity constraints: $f \leq c$ pointwise: i.e. for all $e \in E$,
   $$f(e) \leq c(e).$$

Assume that non-negative quantities $p_s$, for $s \in S$, and $q_t$, for $t \in T$, are given. The goal of this problem is to determine if a valid flow exists such that for all $s \in S$:

$$\sum_{e \in \delta^+(s)} f(e) - \sum_{e \in \delta^-(s)} f(e) = p_s$$

(in words, the source $s$ “produces” $p_s$ units of flow) and such that for all $t \in T$:

$$\sum_{e \in \delta^-(t)} f(e) - \sum_{e \in \delta^+(t)} f(e) = q_t$$

(in words, the sink $t$ “consumes” $q_t$ units of flow).

Use Network Flows to give a polynomial-time algorithm for this decision problem (the answer is YES or NO).