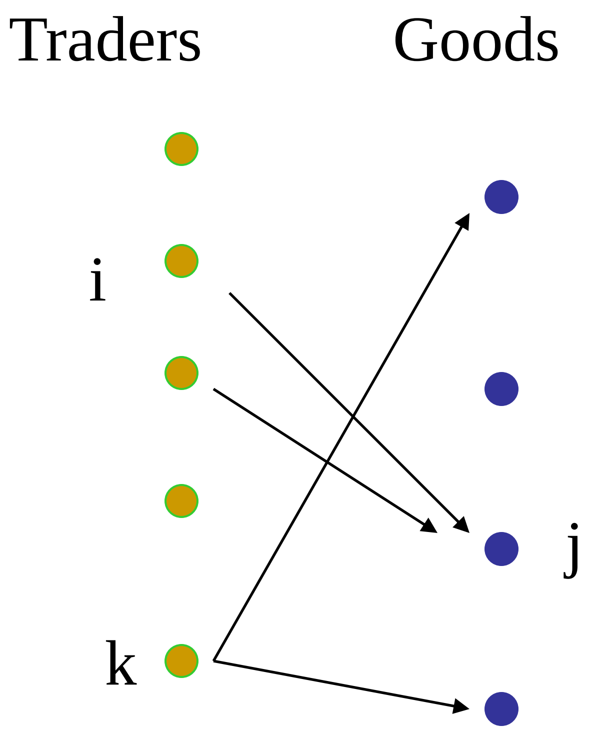
Algorithms: Economic Behavior, Network Games etc.

Sanjiv Kapoor (kapoor@iit.edu)
Illionis Institute of Technology

Market Equilibrium

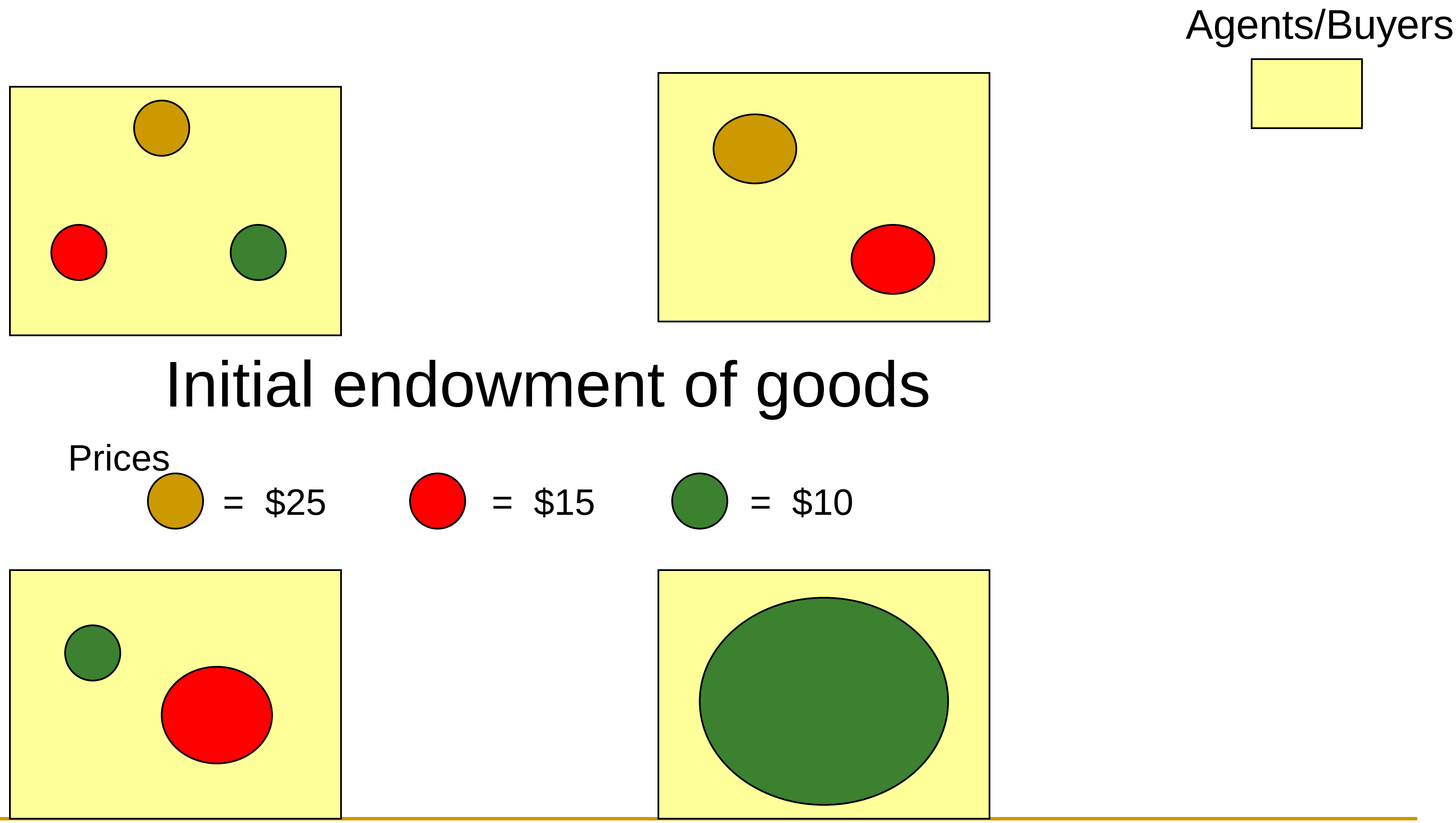
- The market equilibrium problem
- History

Market Model (separable utilities)

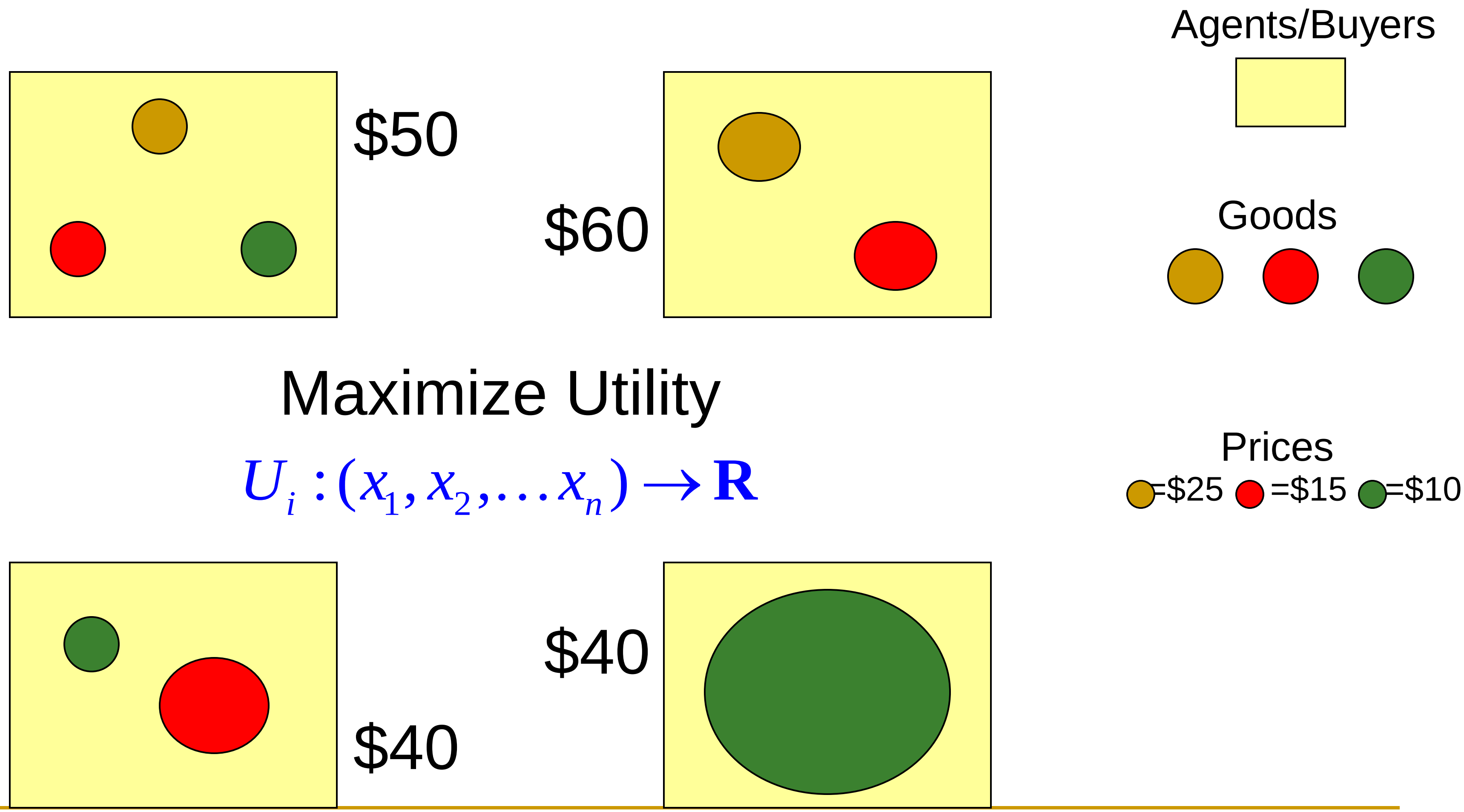


- n traders and m goods
- Each trader has initial endowments of money or goods
  a_{ij} = amount of commodity j with trader I
- Trader I has a utility for good j.

Walras Market Model

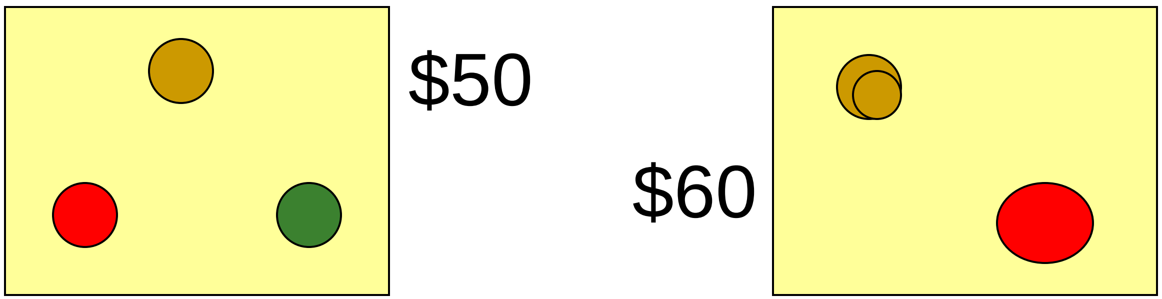


Market: Model



$$U_i : (x_1, x_2, \ldots x_n) \rightarrow \mathbf{R}$$

Market: Model



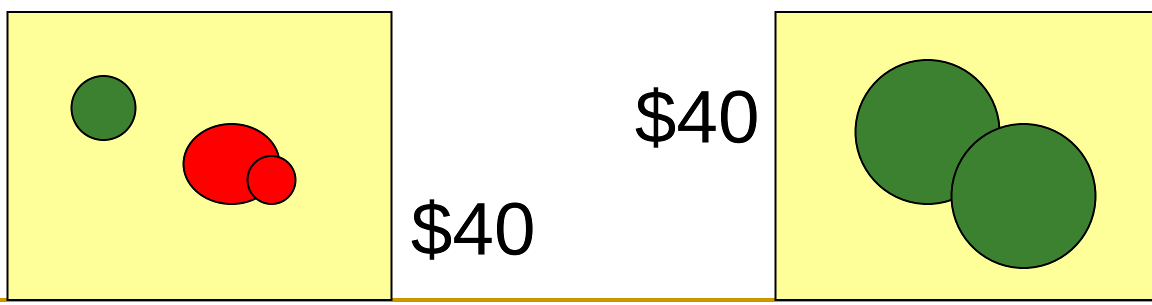
Agents/Buyers

Goods

Find prices so that Demand = Supply

Prices

=$25 =$15 =$10



Maximize Utility

$U_i : (x_1, \ldots x_n) \rightarrow \mathbf{R}$

The General Market Model (Walras)

- n traders and m goods
- Each trader has initial endowments of goods
  a_{ij} = amount of commodity j with trader i
- Traders have utilities on commodity bundles
  $u_i: R_+^m \rightarrow R_+$.
- The market allows traders to exchange commodities
- Each trader acts independently to maximize its own happiness
- Market equilibrium achieved when there is no incentive to trade

Mathematical formulation: Market Equilibrium

- Commodities are divisible
- x_{ij}: the amount of commodity j with trader i after the trade
- Commodity j has price p_j
- No excess or deficiency of any commodity

$$\forall j : \sum_{i=1}^{n} x_{ij} = \sum_{i=1}^{n} a_{ij}$$

$$\forall i : \sum_{j=1}^{m} x_{ij} p_j = \sum_{j=1}^{m} a_{ij} p_j$$

$$\text{Maximize} : u_i(x_{ij})$$

$$\text{Subject to} : \sum_{j=1}^{m} x_{ij} p_j = \sum_{j=1}^{m} a_{ij} p_j$$

The Fisher's Market Model

- Special case of the Walrasian Model
- There are n buyers and m goods
- Buyers have only money
  (buyer i has e_i units of money)
- Goods must be all sold, buyers maximize their bang/buck.

Market Equilibrium History

- Posed by
  - 1891 Fisher
  - 1894 Walras (Walrasian Equilibrium)
- Existence
  - 1954 Arrow and Debreu
- Computation
  - Hydraulic apparatus by Fisher
  - Tatonnement process (Walras)
    - Does it converge?
  - Arrow et al. 1959
    - Stability of a local greedy price adjustment method for "Gross Substitute" utility functions

Computation of Market Equilibrium

- Eisenberg and Gale, 1959
  - Fisher model, additive linear utilities
  - Reduced the problem to a convex optimization problem
- Eaves, 1976
  - Linear complementarity problem
  - Lemke’s algorithm
- Newman and Primak, 1992
  - Ellipsoid method – polynomial-time method

Computation of Market Equilibrium

- Devanur , Papadimitriou, Saberi, Vazirani, 2002
  - Fisher model, separable additive and linear utilities
  - Combinatorial algorithm based on max flows Complexity: n^4/ε max-flow computations ~ n^7/ε
- Jain et al 2003, Devanur and Vazirani 2003
  - Approximation algorithm for Walrasian model, linear utilities
- Jain, 2004
  - General Walrasian model, additive linear utilities, uses Ellipsoid method (inequalities surprisingly similar to Eisenberg and Gale)
- Ye 2004
  - Interior Point method , $O(n^4 L)$

Computation of Market Equilibrium

- Auction Algorithm [GK04a]
  - Linear utilities, Walrasian model
  - Separable Gross Substitutes [GKV04b]
  - Auction Algorithms for Production [KMV05]
  - More General Gross-Substitute Functions [GK06]
- Exact Algorithms for Fisher Model [GK07]
- Algorithms for Resource Allocation markets [FGKKS08]
- Auction for general production models [KS09]

Techniques Used

- Flow based
  - Combinatorial, slow
- Reduction to convex optimization problems
  - Solves for a large class of utility functions
  - Little Economic interpretation
- Interior point method
  - Similar to convex optimization reductions
- Greedy methods based on convex optimization problem
- Auction based
  - Simple and distributed
  - Fast and intuitive
  - Economic interpretation
  - Approximate
  - Need to be extended to more general models

Auction Algorithms for Market Equilibrium [GK04]

- General Walrasian model
- Additive linear utilities
- Approximation algorithm
- Decentralized and distributed
- Simple
- Natural auction interpretation
- Complexity: $1/\varepsilon\, n^3 \log v_{max}$ steps

Price Rollbacks and Path Auctions

- Fisher Model
- Linear utilities
- $(1+\varepsilon)$ approximation
  - $O(n^3 + n^2 \log M) \log(1/\varepsilon))$ time
- Solution is rational
- Exact solution in $O((n^3 + n^2 \log M) L)$ time
- Relation to max flows

The Market Equilibrium Problem

- Linear, additive utilities: $$u_i(x) = \sum_{j=1}^{m} v_{ij} x_{ij}$$
- A family of linear programs
- x_{ij} is a solution to:

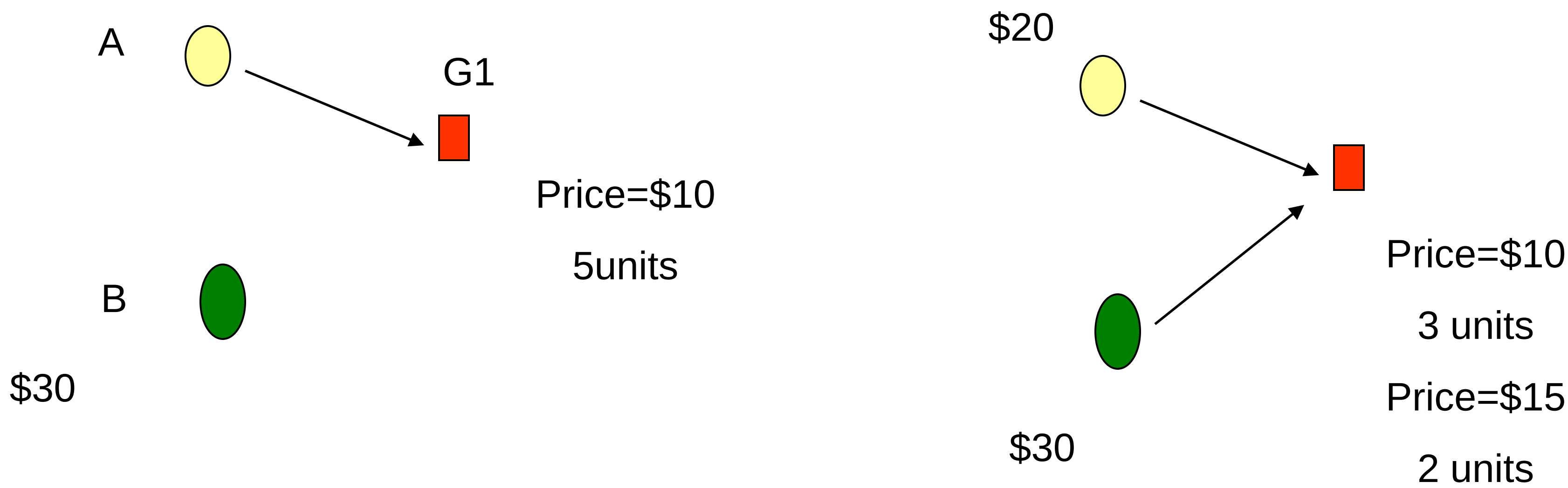
$$\text{Maximize}: \sum_{j=1}^{m} v_{ij} x_{ij}$$

$$\text{Subject to}: \sum_{j=1}^{m} x_{ij} p_j \leq e_i$$

- Markets clear: $$\forall j: \sum_{i=1}^{n} x_{ij} = a_j$$

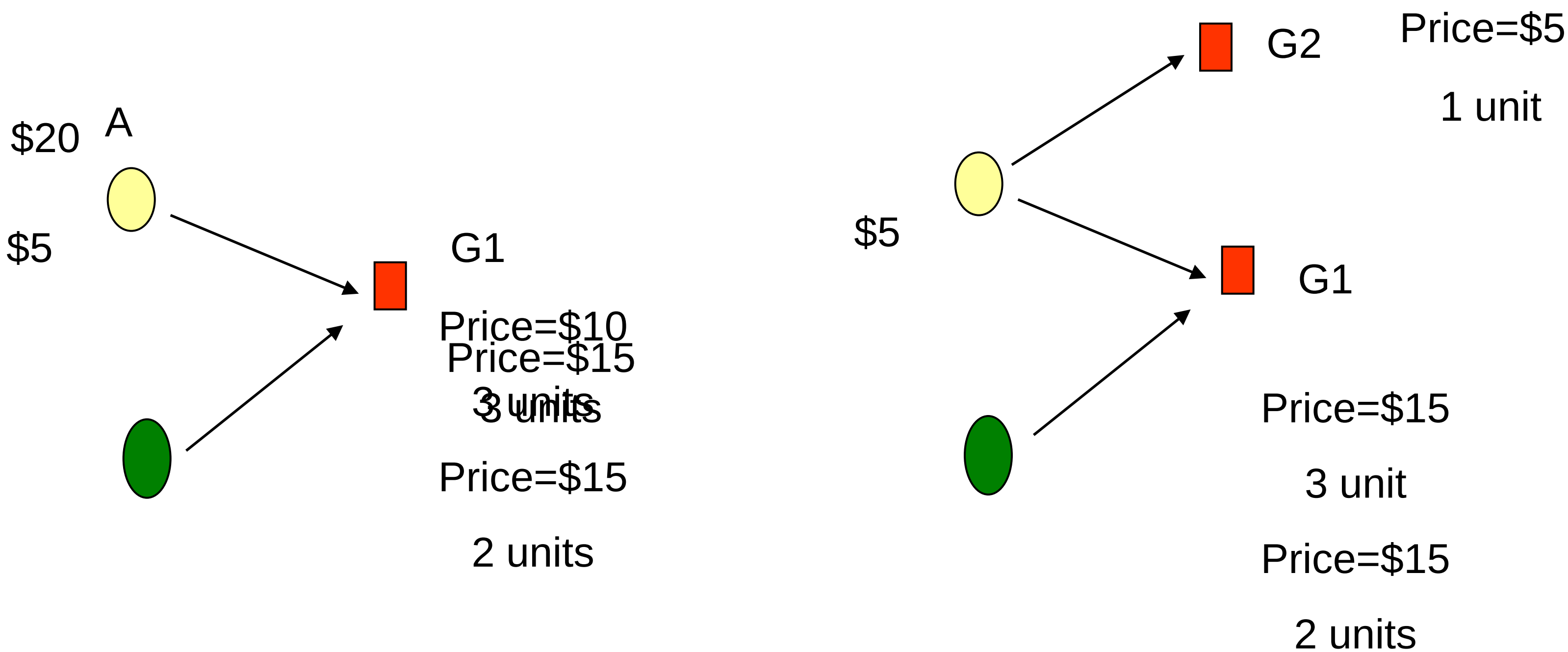
$$\forall i: \sum_{j=1}^{m} x_{ij} p_j = e_i$$

Auction Mechanism



- G1 has maximum utility/price for A upto $15
- G1 has maximum utility/price for B

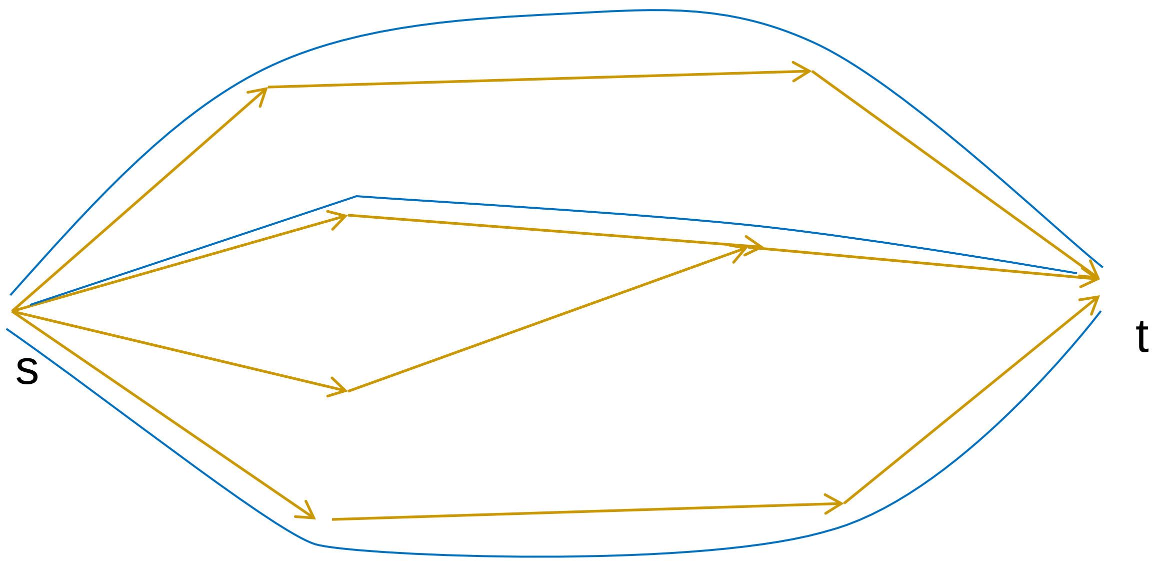
Auction Mechanism



- G1 is maximum utility/price for A upto $15 after which G2 has max utility/price for A

Network Optimization and Games

- Multiple Path Network Routing

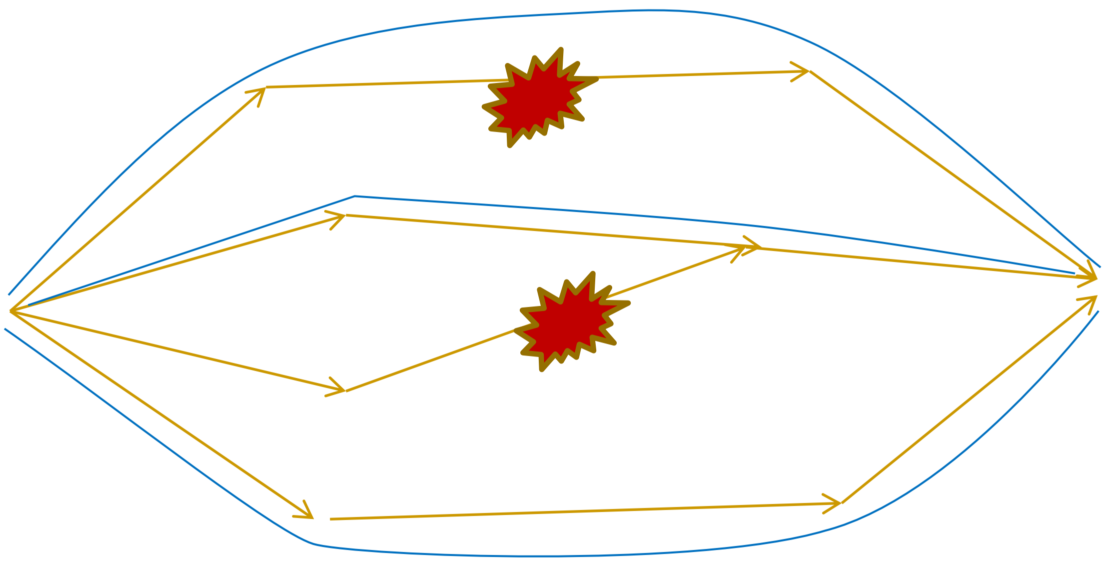


Issues

- Routing Delays
- Requirement of Buffers at End/Intermediate Nodes
- Recent Work-
  - Multipath Routing with Bounded Buffers (INFOCOM 2010)
  - Multiple Source Sink Flows with Bounded Buffers (ICC 2010)

Network Games

Multiple Path Network Routing



Can be modeled as a 2-player Game (Zero-Sum)

- ABSTRACT FILE SYSTEMS

- Users have a large amount of personal files/e-mails due to increased storage space availability for a fraction of the cost.

- Most widely used organizational structure : Hierarchical-Tree

- Limitations
  - A file is accessed by a unique address known as the file path.
  - Organizing is done by using directories, sub-directories, and filenames with extension.
  - It is not very flexible

The Concept

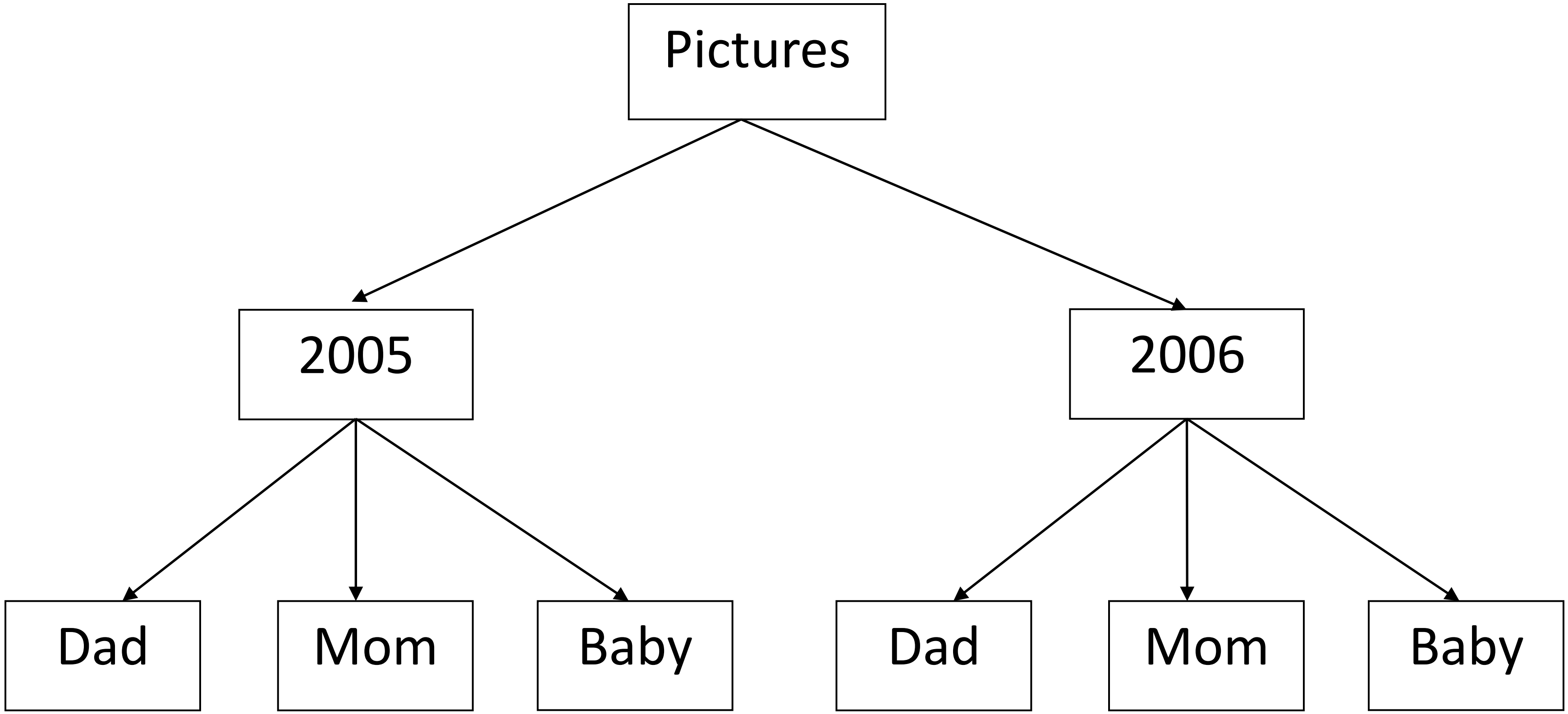
- A methodology to extend the file organization into a **user-defined, Multi-hierarchy - Abstract File System**

Example …

- Consider the following structure of files

Pictures/2006/dad
Pictures/2005/dad
Pictures/2006/mom
Pictures/2005/mom
Pictures/2006/baby
Pictures/2005/baby

Example …



Example …

- Suppose we wanted to access all files which involve dad, i.e.

  Pictures/dad

- The number of files may be substantial and so it is desired that these may be classified further as

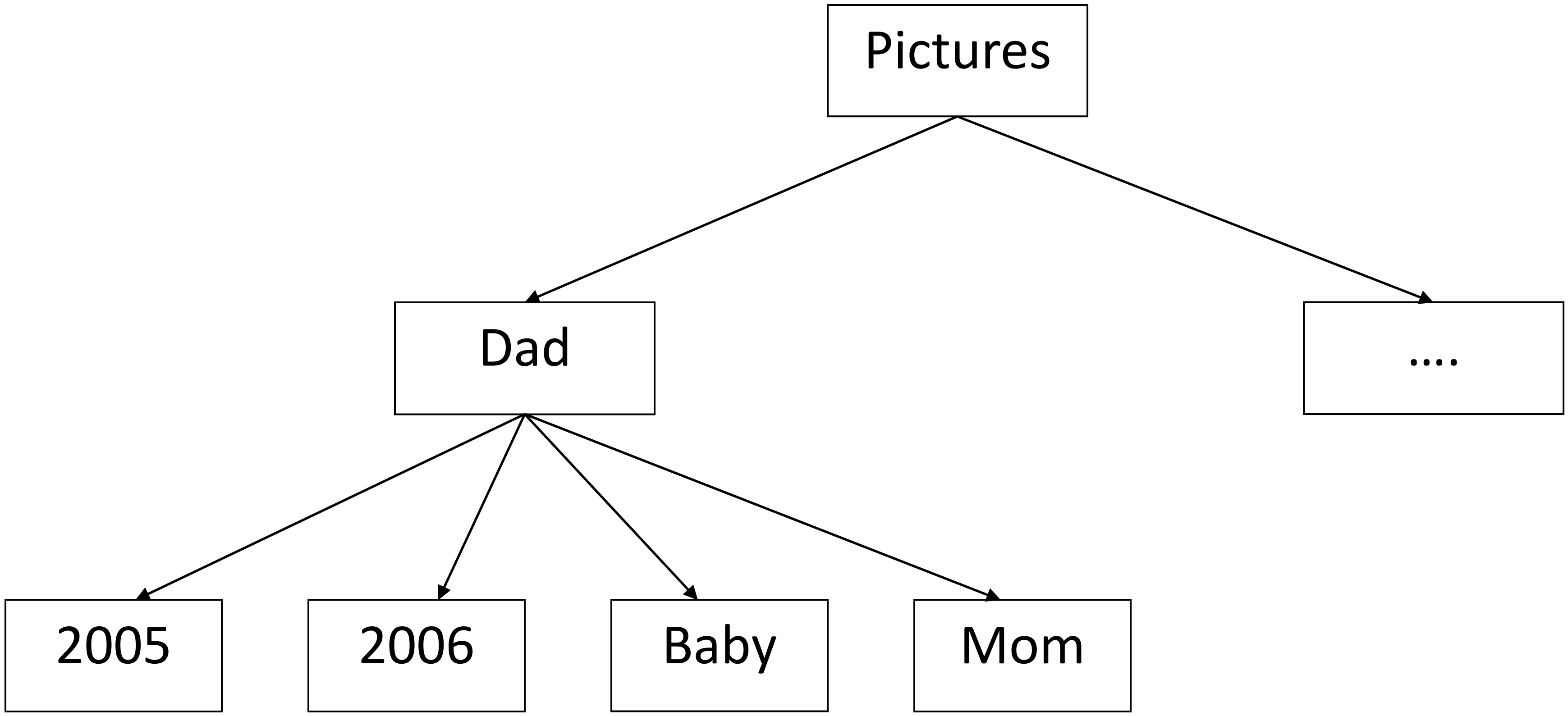
  Pictures/dad/2005

  Pictures/dad/2006

  Pictures/dad/baby

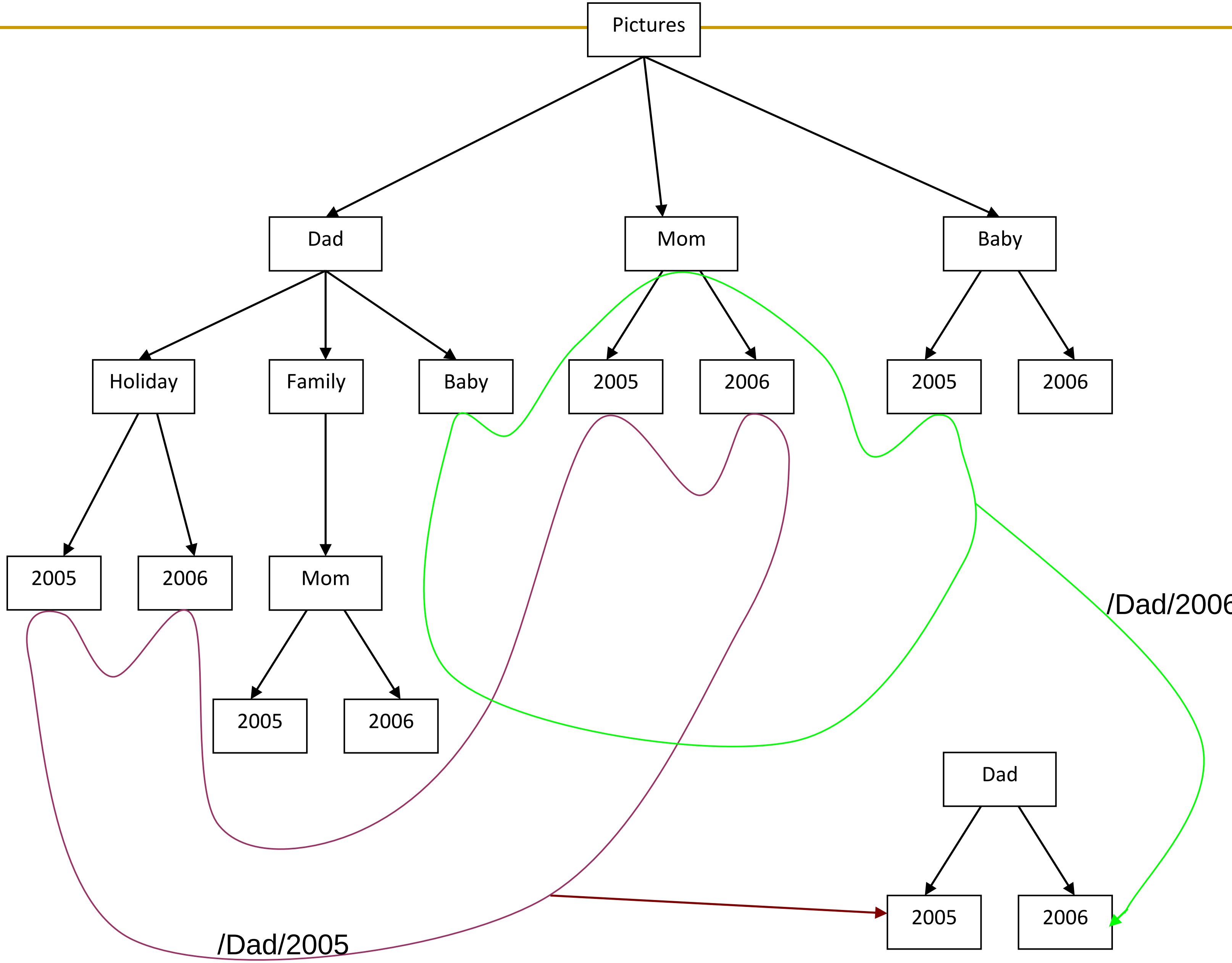
  Pictures/dad/mom

Logically Tagged File Structure



Structured Keywords & Abstract Directories

- We create the notion of **structured keywords and abstract directories** so that files can be organized in any user specified hierarchies.
- This is akin to a hyper-edge labeled with a hierarchical meta-label (Figure below)


Pictures
Dad
Mom
Baby
Holiday
Family
Baby
2005
2006
2005
2006
2005
2006
2005
2006
Mom
2005
2006
/Dad/2006
Dad
2005
2006
/Dad/2005