Algorithms: Economic Behavior, Network Games etc.

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Market Equilibrium

- The market equilibrium problem
- History
Market Model (separable utilities)

- $n$ traders and $m$ goods
- Each trader has initial endowments of money or goods
  - $a_{ij} = \text{amount of commodity } j \text{ with trader } I$
- Trader $I$ has a utility for good $j$. 
Walras Market Model

Initial endowment of goods

Prices:
- Gold = $25
- Red = $15
- Green = $10
Market: Model

Maximize Utility

\[ U_i : (x_1, x_2, \ldots x_n) \rightarrow \mathbb{R} \]
Market: Model

Find prices so that Demand = Supply

Maximize Utility

\[ U_i : (x_1, \ldots, x_n) \rightarrow \mathbb{R} \]
The General Market Model (Walras)

- n traders and m goods
- Each trader has initial endowments of goods $a_{ij} =$ amount of commodity j with trader i
- Traders have utilities on commodity bundles $u_i : R_+^m \rightarrow R_+$. 
- The market allows traders to exchange commodities
- Each trader acts independently to maximize its own happiness
- Market equilibrium achieved when there is no incentive to trade
Mathematical formulation: Market Equilibrium

- Commodities are divisible
- $x_{ij}$: the amount of commodity $j$ with trader $i$ after the trade
- Commodity $j$ has price $p_j$
- No excess or deficiency of any commodity

$$\forall j:\sum_{i=1}^{n} x_{ij} = \sum_{i=1}^{n} a_{ij}$$

$$\forall i:\sum_{j=1}^{m} x_{ij} p_j = \sum_{j=1}^{m} a_{ij} p_j$$

Maximize: $u_i(x_{ij})$

Subject to: $\sum_{j=1}^{m} x_{ij} p_j = \sum_{j=1}^{m} a_{ij} p_j$
The Fisher’s Market Model

- Special case of the Walrasian Model
- There are \( n \) buyers and \( m \) goods
- Buyers have only money
  (buyer \( i \) has \( e_i \) units of money)
- Goods must be all sold, buyers maximize their bang/buck.
Market Equilibrium History

- Posed by
  - 1891 Fisher
  - 1894 Walras (Walrasian Equilibrium)

- Existence
  - 1954 Arrow and Debreu

- Computation
  - Hydraulic apparatus by Fisher
  - Tatonnement process (Walras)
    - Does it converge?
  - Arrow et al. 1959
    - Stability of a local greedy price adjustment method for “Gross Substitute” utility functions
Computation of Market Equilibrium

- Eisenberg and Gale, 1959
  - Fisher model, additive linear utilities
  - Reduced the problem to a convex optimization problem
- Eaves, 1976
  - Linear complementarity problem
  - Lemke’s algorithm
- Newman and Primak, 1992
  - Ellipsoid method – polynomial-time method
Computation of Market Equilibrium

- Devanur, Papadimitriou, Saberi, Vazirani, 2002
  - Fisher model, separable additive and linear utilities
  - Combinatorial algorithm based on max flows Complexity: \( \frac{n^4}{\epsilon} \) max-flow computations \( \sim \) \( \frac{n^7}{\epsilon} \)

- Jain et al 2003, Devanur and Vazirani 2003
  - Approximation algorithm for Walrasian model, linear utilities

- Jain, 2004
  - General Walrasian model, additive linear utilities, uses Ellipsoid method (inequalities surprisingly similar to Eisenberg and Gale)

- Ye 2004
  - Interior Point method, \( O(n^4 L) \)
Computation of Market Equilibrium

- **Auction Algorithm [GK04a]**
  - Linear utilities, Walrasian model
  - Separable Gross Substitutes [GKV04b]
  - Auction Algorithms for Production [KMV05]
  - More General Gross-Substitute Functions [GK06]
- **Exact Algorithms for Fisher Model [GK07]**
- **Algorithms for Resource Allocation markets [FGKKS08]**
- **Auction for general production models [KS09]**
Techniques Used

- Flow based
  - Combinatorial, slow
- Reduction to convex optimization problems
  - Solves for a large class of utility functions
  - Little Economic interpretation
- Interior point method
  - Similar to convex optimization reductions
- Greedy methods based on convex optimization problem
- Auction based
  - Simple and distributed
  - Fast and intuitive
  - Economic interpretation
  - Approximate
  - Need to be extended to more general models
Auction Algorithms for Market Equilibrium [GK04]

- General Walrasian model
- Additive linear utilities
- Approximation algorithm
- Decentralized and distributed
- Simple
- Natural auction interpretation
- Complexity: $1/\varepsilon n^3 \log v_{\text{max}}$ steps
Price Rollbacks and Path Auctions

- Fisher Model
- Linear utilities
- $(1+\varepsilon)$ approximation
  - $O(n^3 + n^2\log M) \log(1/\varepsilon)$ time
- Solution is rational
- Exact solution in $O((n^3 + n^2\log M) L)$ time
- Relation to max flows
The Market Equilibrium Problem

- Linear, additive utilities: 
  \[ u_i(x) = \sum_{j=1}^{m} v_{ij} x_{ij} \]

- A family of linear programs

- \( x_{ij} \) is a solution to:

- Markets clear:
  \[ \forall j: \sum_{i=1}^{n} x_{ij} = a_j \]
  \[ \forall i: \sum_{j=1}^{m} x_{ij}p_j = e_i \]
- G1 has maximum utility/price for A upto $15
- G1 has maximum utility/price for B
G1 is maximum utility/price for A upto $15 after which G2 has max utility/price for A.
Network Optimization and Games

- Multiple Path Network Routing
Issues

- Routing Delays
- Requirement of Buffers at End/Intermediate Nodes

Recent Work-
- Multipath Routing with Bounded Buffers (INFOCOM 2010)
- Multiple Source Sink Flows with Bounded Buffers (ICC 2010)
Network Games

Multiple Path Network Routing

Can be modeled as a 2-player Game (Zero-Sum)
ABSTRACT FILE SYSTEMS

- Users have a large amount of personal files/e-mails due to increased storage space availability for a fraction of the cost.

- Most widely used organizational structure: Hierarchical-Tree
Limitations

- A file is accessed by a unique address known as the file path.
- Organizing is done by using directories, sub-directories, and filenames with extension.
- It is not very flexible
A methodology to extend the file organization into a user-defined, Multi-hierarchy - Abstract File System
Consider the following structure of files

- Pictures/2006/dad
- Pictures/2005/dad
- Pictures/2006/mom
- Pictures/2005/mom
- Pictures/2006/baby
- Pictures/2005/baby
Example ...

![Diagram with categories](example_diagram.png)
Example …

- Suppose we wanted to access all files which involve dad, i.e.
  
  Pictures/dad

- The number of files may be substantial and so it is desired that these may be classified further as
  
  Pictures/dad/2005
  Pictures/dad/2006
  Pictures/dad/baby
  Pictures/dad/mom
Logically Tagged File Structure

- Pictures
  - Dad
    - 2005
    - 2006
    - Baby
    - Mom
  - ....
Structured Keywords & Abstract Directories

- We create the notion of **structured keywords and abstract directories** so that files can be organized in any user specified hierarchies.
- This is akin to a hyper-edge labeled with a hierarchical meta-label (Figure below)