Cops and Robber Pursuit

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Cops and Robber—Assumptions

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- Cops know nothing of the robber's position
- Cops and the robber have the same maximum speed and move continuously in a continuous region.
- What is the maximum number of cops that a robber can evade?

Current Best Lower Bound

Theorem

If there are fewer than $\lfloor n/5.889 \rfloor$ *cops, the robber can forever evade capture on the continuous* $n \times n$ *square.*

A Better Bound

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$\lfloor n/\sqrt{5} \rfloor + 1$ cops can capture a robber on the continuous $n \times n$ square.

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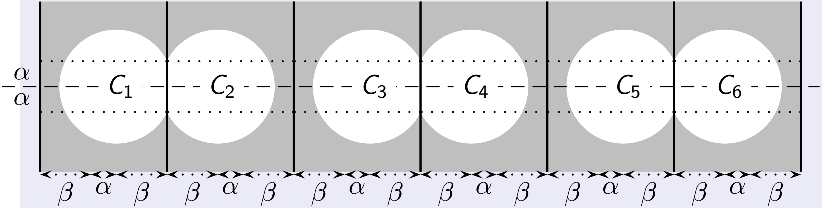
Proof.



Maximizing $\alpha + 2\beta$ subject to $\beta^2 + \alpha^2 < 1$ gives $\alpha = 1/\sqrt{5}$ and $\beta = 2/\sqrt{5}$ so that $\alpha + 2\beta = \sqrt{5}$. □

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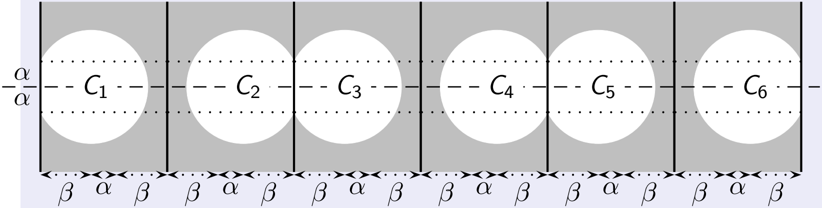

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How Powerful is One Cop?

Theorem

If there is only a single cop, the robber can forever evade capture in the continuous 4.5×4.5 *square.*

One Cop–Upper Bound

Theorem

A single cop can capture a robber in a square with side length less than $\ell = 3\sqrt{2}/2 = 2.121\cdots$.

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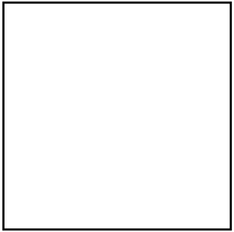

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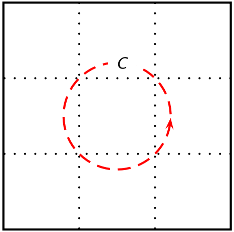

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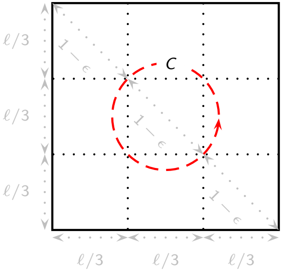

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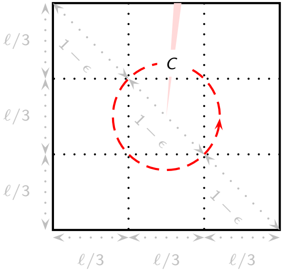

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