Introduction

Capacitors are used in timing circuits in many devices. The time that your dome lights inside your car stay on after you turn off your cars ignition at night is one example of how a capacitor can be used to maintain the lighting long enough for you to remove the keys and collect your things before exiting. The value we use to characterize these kinds of circuits is given by the time constant defined as: \( \tau = RC \), where \( R \) is the circuit resistance (your dome light in this case) and \( C \) is the capacitance, in Farads (F). In this lab, we will measure the time constant of a capacitor in two situations - one where the time constant is several seconds long, and the other for time constants on the order of several milliseconds long. To accomplish this, we construct a RC circuit, which contains a power supply (DC then AC), a resistor \( R \), and of course a capacitor \( C \) (see Figure 1). If at time \( t = 0 \) the Switch A is closed (Switch B remains open), charges will begin to build up in the capacitor. These charges do not accumulate within the capacitor instantaneously due to the “resistance” provided by the resistors. The potential difference across the capacitor can be expressed as

\[
V(t) = V_0 \left(1 - e^{-t/\tau}\right) \tag{1}
\]

where \( \tau = RC \) is defined as the time constant of the RC circuit, and \( V_0 \) is the maximum potential difference across the capacitor. After a sufficiently long time (much larger than the time constant), if Switch A is opened while Switch B is closed, the capacitor will discharge all of its accumulated charges. The potential difference across the capacitor for this process can be expressed as

\[
V(t) = V_0 e^{-t/\tau} \tag{2}
\]

Figure 1: Circuit for RC charge-discharge measurement where \( V(t) \) is the sensor used to measure the potential difference across the capacitor as a function of time.

The time dependence of the potential difference \( V(t) \) for the charging and discharging process is shown in Figure 2. The time constant can be determined by observing either the charging or discharging process. For the charging process, \( \tau \) is equal to the time for \( V(t) \) to reach 63% of its “final” value. For the discharging process, \( \tau \) is equal to the time for \( V(t) \) to fall to 63% from its initial value. These values can actually be measured at any time during the charging or discharging cycle, as long as one waits long enough for the capacitor voltage to increase or decrease by 63% of a measured value. If one can capture the voltage passing 10V during a discharge cycle, then one only needs to measure the time it takes for the voltage to decrease by 6.3V to 3.7V (a 63% decrease). You will practice this latter approach with the next exercise.
Part 1 - Measurement of a Long Time Constant:

In this experiment, you will measure $V(t)$ across the capacitor as it discharges.

First measure the capacitance of the very large capacitor provided (the nominal value is 47 $\mu$F). Construct the circuit as shown in Figure 3, making sure the electrolytic capacitor is connected with correct polarity. Notice that when the capacitor is connected to the battery, current will flow until the capacitor is completely charged. When the battery is disconnected, the capacitor discharges through your multimeter. We will use the PASCO Capstone voltage sensor (which will used as a digital voltmeter with a computer interface) contains an internal resistor $R_i$. The voltmeter acts as the “load” resistor for the circuit, as well as a measuring device. In other words, the resistor in the PASCO Capstone system is the R in the RC circuit. The internal resistor is $R_i = 1 \text{ M}\Omega$ and can be measured using Ohm’s law. Before taking any measurements, set the digital voltmeter to the appropriate settings - a voltage scale around 12 Volts, and set the voltmeter sample rate to 1 Hz (1 measurement per second).

Charge the capacitor by connecting it to the two 6V batteries. Make sure to hook them up in series so that you are applying 12V to your capacitor. This should take only a few seconds for the voltage reading to go beyond 10 V. Disconnect the batteries and start time ($t = 0$) when the voltmeter reads exactly 10 V (i.e. $V_o = 10$V). Record the subsequent times when the voltage reads 9 V, 8 V, 7 V, ... etc. Depending on how slow the decay rate is, you may not want to wait for it to reach 1 V. Record the time it takes to reach 3V at a minimum. Note that the first few readings may go too fast. If so, you may have to recharge the capacitor and then find the time just for it to discharge from 10 V to 8 V, for instance. Repeat as often as necessary. Create a two-column data table. The first column for time and the second column for $V(t)$. Be sure to record the values of capacitance and resistance (C and R) for use later.

Answer the following questions in your lab report:

1. Equation 2 can be written as $\ln V(t) = \ln V_o - t/\tau$. This means that if we plot $\ln V(t)$ versus $t$, the slope will correspond to $-1/\tau$. Find the natural logarithm of $V(t)$ from your data and plot $\ln V(t)$ versus $t$.

2. Find the slope of the best-fit line and thus obtain the experimental value of the time constant. Compare the time constant obtained from this slope with the value of $\tau$ obtained using $\tau = RC$ and the values of $R_i$ and C obtained previously.
Part 2 - Measurement of a Short Time Constant

In this part, we measure the short time constant of another RC circuit by continuously charging and discharging the capacitor. We accomplish this by connecting the RC combination to a power supply (function generator) producing a square wave voltage pattern as shown in Figure 4. During the period when the applied voltage is $V$, the capacitor is charging, whereas during the period when the applied voltage is $-V$, the capacitor is discharging. Since this charging and discharging processes occur very rapidly, a convenient way to study these processes is using the oscilloscope since time scales are now milliseconds rather than seconds.

**Part 2 Procedure:**

1. Using the Capacimeter and an Ohmmeter, find the capacitance and resistance of the small capacitor and the resistor given for this part (nominal values $10\ \text{nF}$ and $12\ \text{k}\Omega$, respectively). Connect the RC circuit as shown in Figure 5. Make sure you connect the input of the oscilloscopes CH1 to the point between the capacitor and resistor in the circuit.
2. Switch on the function generator and set it to 1.3 kHz. Turn on the digital oscilloscope and adjust the vertical and horizontal positioning knob, the time/div scale, and the V/div scale for Channel 1 until you obtain the charging/discharging trace. Press the autoset button on the upper right of the oscilloscope if it takes you more than a few minutes to get a display. Expand the trace and adjust the amplitude of the function generators square wave amplitude so that it extends across the whole 8 divisions of the screen, only making visible one or two complete periods of the square wave. The screen on your oscilloscope should look similar to the red dashed line of Figure 4.

3. Now record the time (t) it takes for the voltage of the capacitor to reach 63% of the highest voltage. Similarly, record the time when the discharging voltage decreases 63% from its highest voltage. These two values should be roughly identical. Find the average and use this as the experimental time constant.

4. Plot the oscilloscope trace and include it in your laboratory report. Illustrate in your sketch how you obtained the time constant and the values you used.

5. Monitor the voltage applied from the function generator using the oscilloscope probe and/or any other provided cables (via Ch 2). Use the autoset function on the oscilloscope and see if you can reproduce the display shown in Figure 4. This will require moving one of the channels vertically to align it with the other channel. The amplitudes should match. Switch to 4.0 kHz for the frequency of the function generator and note what happens to the voltage across the capacitor.

![Figure 5: Setup for measurement of fast time constant.](image)

Answer the following questions in your lab report:

1. Compare your measured value with the product of RC obtained from the individual values of R and C measured earlier and equation $\tau = RC$. 
2. Using Equation 2, show the mathematical reasoning behind why the time constant, $\tau$, represents a 63% decrease in the initial voltage for a discharging capacitor.