Experiment 10: Mass to Charge Ratio

Introduction

Just as moving charges induce magnetic fields, as seen in the previous lab, charged particles moving in the presence of a magnetic field feel a magnetic force. A particle with charge \( q \) moving with velocity \( v \) in a magnetic field \( B \) will experience a force \( F \) perpendicular to its direction of motion described by the Lorentz force equation

\[
F = qv \times B
\]  

(1)

If a uniform magnetic field \( B \) is perpendicular to the initial direction of motion of an electron beam, the electrons will be deflected by a force that is always perpendicular to their velocity and to the magnetic field \( B \). Consequently, the beam will be deflected into a circular trajectory with radius \( r \), as shown in Figure 1.

![Figure 1: Path of electron in a uniform magnetic field. The magnetic field points into the page. Although \( \mathbf{v} \times \mathbf{B} \) points outward from the center of the circle, the negative electron feels a centripetal force resulting in uniform circular motion.](image)

If we simply observe that the electron moves uniformly in a circular path, we can conclude that there is a constant centripetal force \( F_c \) which pulls the electron toward the center of the circle, that is

\[
F_c = \frac{m_e v^2}{r}
\]

(2)

where \( m_e = 9.11 \times 10^{-31} \) kg, the mass of an electron. The origin of this centripetal force is the Lorentz force, thus Equations 1 and 2 are equal to each other, i.e.

\[
evB = \frac{m_e v^2}{r}
\]

(3)

where \( e \) is the charge of an electron \((e = -1.6 \times 10^{-19} \) C). Here we have used the condition that \( \mathbf{v} \) is perpendicular to \( \mathbf{B} \) in this case. Simplifying Equation 3, we obtain the ratio \( e/m \) for the electron as

\[
\frac{e}{m_e} = \frac{v}{rB}
\]

(4)
In this experiment, the electron originates from the cathode with a negligible initial velocity. When a potential difference $V$ is applied, its speed will increase such that when it reaches the anode, it will have gained a kinetic energy equal to the potential energy $eV$ ($\frac{1}{2}mv^2 = eV$). The velocity is given as

$$v = \sqrt{\frac{2eV}{m_e}} \quad (5)$$

Substituting Equation 5 into Equation 4, we obtain

$$\frac{e}{m_e} = \frac{2V}{r^2B^2} \quad (6)$$

The magnetic field for our experiment is provided by a Helmholtz coil configuration, similar to the one from the previous experiment. Recall that for a Helmholtz coil with $N$ turns, radius $R$, and current $I$, the magnetic field strength at the center of the configuration is given by

$$B = \frac{8\mu_0IN}{5\sqrt{5}R} \quad (7)$$

Substituting this expression for the field into Equation 6, we finally have

$$\frac{e}{m_e} = \frac{125}{32} \frac{VR^2}{\mu_0^2I^2N^2r^2} \quad (8)$$

The electron beam will be generated within an “e/m tube”, which is a spherical glass vacuum tube, containing an “electron gun” and deflection plates, as shown in Figure 2a. The electron gun consists of a cathode filament in the tube which is heated to high temperatures emitting electrons. The filament is surrounded by a cylindrical anode at a potential $V$ relative to the cathode (see Figure 2b). This accelerates the electrons and the ones that escape through the opening of the cylinder will have a kinetic energy of $eV$. The electrons are accelerated between the cathode and the anode. These electrons will experience the uniform magnetic field of the Helmholtz coil and will move in a circular path shown in the figure. To be able to see the path taken by the electrons, the tube is filled with helium at very low pressure. This allows us to see a luminescent trace of the electrons path because some of the electrons will collide with the helium atoms, which become excited and thus radiate visible light. By varying the current going through the Helmholtz coil, we can vary the magnetic field and thus the magnetic force. This in turn will vary the radius of the circular path on which the electrons travel.

We will use an adjustable mirrored scale to measure the radius of the electron beam path without parallax error. The mirrored scale is attached to the back of the rear Helmholtz coil and is illuminated by lights that automatically light up when the heater of the electron gun is turned on. The mirrored scaled can be moved up and down by adjusting the knobs (thumbscrews) on its side - the ruler scale, adjacent to the knobs, will tell you if the mirror scale is leveled. Make sure the scale is centered behind the tube.

To carefully measure the radius of the electron beam, look through the tube and avoid parallax errors by moving your head to align the electron beam with the reflection of the beam that you can see on the mirrored scale. Measure the radius of the beam as you see it on both sides of the scale, then average the results. Use the cloth hood to reduce any light pollution.
Procedure:

In this lab we will observe the electron beam trajectory with respect to the magnetic field of the Helmholtz coils and the applied potential accelerating the electrons. We will then determine the electron’s charge to mass ratio \(\frac{e}{m}\) experimentally and compare it to theoretical calculations using Equation 8.

Warning: Do not exceed 6.0 Volts in the electron gun (heater) circuit of the e/m apparatus.

1. Measure the radius of the Helmholtz coil \((R)\). Find the number of turns, \(N\).

2. The Helmholtz coil and the tube should already be connected to their respective power supply/circuits. Identify the internal ammeter that shows the current through the Helmholtz coil (this is \(I\) in Equation 8), and the external digital voltmeter that shows the potential difference between the cathode and the anode (this is \(V\) in Equation 8). In this experiment, \(V\) ranges from 150 Volts to 300 Volts. Before switching on the power supply unit(s), make sure all the current and voltage knobs are at their minimum values.

3. Switch on the power supply for the tube and adjust the power supply of the “heater” filament to 5.0 Volts. This will supply power to both the electrons and the Helmholtz coil. A finer beam is obtained if you lower the filament power after you have found the blue glow. Once this is done, do not change this setting for the rest of the experiment.

4. Increase the accelerating potential (the potential difference between the anode and cathode) to 150 Volts (read off the DVM). The beam will curve by the Helmholtz coil field. Make sure the beam path is parallel with the coils. If not, you can rotate the tube until it is (ask the TA for help). Knowing the direction of the electron’s path, figure out the direction of the current in the Helmholtz coil using the right hand rule.
5. Adjust the power supply so that the electron beam completes a circular path, passing in front of the mirrored scale, with a radius of 3.5 cm. By lining the electron beam up with its image in the mirrored scale, you can measure the diameter/radius of the beam path. You may want to take measurements of the beam radius as you see it on both sides of the scale, then average the results.

6. Record the value of the radius and carefully read the respective current of the Helmholtz coil from the internal ammeter. You should have a radius and a current value for the 150 V accelerating potential.

7. Keeping the accelerating potential constant at 150V, find and record the respective currents for the following 3 other beam radii: 4.0 cm, 4.5 cm, and 5.0 cm.

8. Repeat steps 5, 6, and 7 for the different potentials: 175 V, 200 V, 225 V, and 250 V. Your data should resemble the table given below, where \( r_i \) are the beam radii used in step 7. Find the respective currents for each radii and its corresponding accelerating potential. Fill in these current values in the data table, accordingly.

<table>
<thead>
<tr>
<th>Beam Radius</th>
<th>150 V</th>
<th>175 V</th>
<th>200 V</th>
<th>225 V</th>
<th>250 V</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r_1 = 3.5 \text{ cm} )</td>
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<tr>
<td>( r_2 = 4.0 \text{ cm} )</td>
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<tr>
<td>( r_3 = 4.5 \text{ cm} )</td>
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<tr>
<td>( r_4 = 5.0 \text{ cm} )</td>
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</tbody>
</table>

A full lab report is not necessary for this lab. Answer the questions on the following page and turn it in with your signed datasheet.
PHYS 221, Lab 10 Questions

Name: ___________________________  CWID: ________________

Write your answers on a separate sheet and attach your signed datasheet when turning it in. You must show all of your work for full credit. Make it clear to me you understand what you’re doing. Any graphs or tables should be made via computer software and attached to this handout.

1. For each radius obtained, plot $I^2$ versus $V$. This means that for each ROW of your data table, one data set is to be plotted. You will end up with four sets of data, representing four different values of $r$. You may plot all four data sets on the same graph if they are clearly labeled.

2. Find the slope of the best-fit line for each data set. What does this slope represent?

3. From Equation 8, the slope of the best-fit line of this graph corresponds to

$$\frac{125}{32} \frac{R^2}{\mu_0^4 N^2} \frac{1}{r^2} \left(\frac{e}{m}\right)$$

Prove this in the theory section of your report.

4. Using Equation 9, find the experimental value of $e/m$ using each of your slope values. Keep in mind that each slope corresponds to a particular $r$ value! [Use a spreadsheet, for instance, to perform this tedious calculation.]

5. Find the average and standard deviation of the experimental value of $e/m$ and compare it with the theoretical value. What are the sources of error?

6. Calculate the velocity of electrons for each accelerating potential.