Illinois Institute of Technology
Physics

M.Sc. Comprehensive and Ph.D. Qualifying Examination

PART I

Thursday, August 23, 2018
4:00 - 8:00 PM

General Instructions

1. Each problem is to be done on a separate booklet. Label the front of each book with the identifying code letter you picked, the part number of the exam, and the number of the problem only; for example: A-I.6. Do not write your name or IIT ID number on any material handed in for grading.

2. Any numerical data not specified in a problem should be found in the table of constants at the front of the exam.

3. DON’T PANIC: It is not expected that each student will completely solve every problem. However, it is advisable to do a thorough job on those problems that you do solve.
Physical Constants

- Speed of light in vacuum: $c = 2.998 \times 10^8$ m/s
- Planck’s constant: $h = 6.626 \times 10^{-34}$ J·s, $\hbar = h/2\pi = 1.055 \times 10^{-34}$ J·s, $= 6.582 \times 10^{-16}$ eV·s
- Permeability constant: $\mu_o = 4\pi \times 10^{-7}$ N/A²
- Permittivity constant: $\frac{1}{4\pi\epsilon_o} = 8.898 \times 10^9$ N·m²/C²
- Fine structure constant: $\alpha = \frac{e^2}{4\pi\epsilon_o\hbar c} = 7.30 \times 10^{-3} \approx \frac{1}{137}$
- Gravitational constant: $G = 6.67 \times 10^{-11}$ m³/s²·kg
- Avogadro’s number: $N_A = 6.023 \times 10^{23}$ mole⁻¹
- Boltzmann’s constant: $k = 1.381 \times 10^{-23}$ J/K = 8.617 × 10⁻⁵ eV/K
- kT at room temperature: $k \cdot 300$ K = 0.0258 eV
- Universal gas constant: $R = 8.314$ J/mole·K
- Stefan-Boltzmann constant: $\sigma = 5.67 \times 10^{-8}$ W/m²·K⁴
- Electron charge magnitude: $e = 1.602 \times 10^{-19}$ C
- Electron rest mass: $m_e = 9.109 \times 10^{-31}$ kg = 0.5110 MeV/c²
- Neutron rest mass: $m_n = 1.675 \times 10^{-27}$ kg = 939.6 MeV/c²
- Proton rest mass: $m_p = 1.672 \times 10^{-27}$ kg = 938.3 MeV/c²
- Deuteron rest mass: $m_d = 3.343 \times 10^{-27}$ kg = 1875.6 MeV/c²
- Atomic mass unit (C₁₂ = 12): $u = 1.661 \times 10^{-27}$ kg = 931.5 MeV/c²
- Mass of earth: $M_E = 5.98 \times 10^{24}$ kg
- Radius of earth: $R_E = 6.37 \times 10^6$ m
- Mass of sun: $M_S = 1.99 \times 10^{30}$ kg
- Radius of sun: $R_S = 6.96 \times 10^8$ m
- Gravitational acceleration at earth’s surface: $g = 9.81$ m/s²
- Atmospheric pressure: $= 1.01 \times 10^5$ N/m²
- Radius of earth’s orbit: $= 1.50 \times 10^{11}$ m
- Radius of moon’s orbit: $= 3.84 \times 10^8$ m

Conversion Factors

- 1 eV = $1.602 \times 10^{-19}$ J
- 1 Å = $10^{-10}$ m
- 1 barn (b) = $10^{-28}$ m²
- 0° Celsius = 273.16 K
- 1 J = $6.242 \times 10^{18}$ eV
- 1 Fermi = $10^{-15}$ m
- 1 in = 2.54 cm
- 1 cal = 4.19 J
Problem 1: A string wraps around a uniform cylinder of mass $M$, which rests on a fixed plane. The string passes up over a massless pulley and is connected to a mass $m$, as shown in the figure. Assume that the cylinder rolls without slipping on the plane and that the string is parallel to the plane. What is the acceleration of the mass $m$? What is the condition on the ratio $M/m$ for which the cylinder accelerates down the plane?

Problem 2: By any method you choose show that the following transformation is canonical:

$$
\begin{align*}
    x &= \frac{1}{\alpha} \left( \sqrt{2} P_1 \sin Q_1 + P_2 \right), \\
    y &= \frac{1}{\alpha} \left( \sqrt{2} P_1 \cos Q_1 + Q_2 \right), \\
    p_x &= \frac{\alpha}{2} \left( \sqrt{2} P_1 \cos Q_1 - Q_2 \right), \\
    p_y &= -\frac{\alpha}{2} \left( \sqrt{2} P_1 \sin Q_1 - P_2 \right),
\end{align*}
$$

where $\alpha$ is some fixed parameter.

Apply this transformation to the problem of a particle of charge $q$ moving in a plane that is perpendicular to a constant magnetic field $B$. Express the Hamiltonian for this problem in the $(Q_i, P_i)$ coordinates, letting the parameter $\alpha$ take the form

$$
\alpha^2 = \frac{qB}{c}.
$$

From this Hamiltonian obtain the motion of the particle as a function of time.

Hint: The Hamiltonian $H(x, y, p_x, p_y) = \frac{1}{2m} \left( p - \frac{q}{c} A \right)^2$, where $A = \frac{B}{2} (-y\hat{i} + x\hat{j})$.

Problem 3: Two identical masses $m$ are constrained to move on a horizontal hoop. Two identical springs with spring constant $k$ connect the masses and wrap around the hoop (see figure). One mass is subject to a driving force $F_d \cos(\omega_d t)$. Find the driven oscillation solution for the motion of the masses.
Problem 4: A physical pendulum consists of a uniform solid disk of mass $M$ and radius $R$ supported in the vertical plane by a pivot located a distance $d < R$ from the center of the disk. The disk is displaced a small angle and released. Derive an expression for the period of the resulting motion.

![Diagram of a physical pendulum](image)

Problem 5: A tall cylindrical vessel with gaseous nitrogen is located in a uniform gravitational field in which the free-fall acceleration is equal to $g$. The temperature of the nitrogen varies along its height $h$ so that its density is the same throughout the volume. Find the temperature gradient $dT/dh$.

Problem 6: Two thermally insulated vessels 1 and 2 are filled with air and connected by a short tube equipped with a valve. The volumes of the vessels, the pressures and temperatures of air in them are known ($V_1, p_1, T_1$ and $V_2, p_2, T_2$). Find the air temperature and pressure established after the opening of the valve.

Problem 7: A material contains many two-molecule-binding sites. If neither is occupied, the energy is 0, if one of the two is occupied, the energy is $\varepsilon$, and if both are occupied, the energy is $+3\varepsilon$. Compute the (classical) partition function, the free energy, the energy, and the entropy as a function of inverse temperature $\beta$.

Problem 8:

(a) Let 580 nm light be normally incident on a double slit system for which $d = 4800$ nm. How many orders (maxima) will be visible on a screen placed in front of the slits?

(b) If the slit width is $a = 800$ nm, which orders, if any, will be missing?