Illinois Institute of Technology
Physics

M.Sc. Comprehensive and Ph.D. Qualifying Examination

PART II

Saturday, August 25, 2018
10:00 AM - 2:00 PM

General Instructions

1. Each problem is to be done on a separate booklet. Label the front of each book with the identifying code letter you picked, the part number of the exam, and the number of the problem only; for example: A-I.6. Do not write your name or IIT ID number on any material handed in for grading.

2. Any numerical data not specified in a problem should be found in the table of constants at the front of the exam.

3. DON’T PANIC: It is not expected that each student will completely solve every problem. However, it is advisable to do a thorough job on those problems that you do solve.
Physical Constants

Speed of light in vacuum  \( c = 2.998 \times 10^8 \text{ m/s} \)

Planck’s constant
\[
\begin{align*}
    h &= 6.626 \times 10^{-34} \text{ J} \cdot \text{s} \\
    \hbar &= h/2\pi \\
    &= 1.055 \times 10^{-34} \text{ J} \cdot \text{s} \\
    &= 6.582 \times 10^{-16} \text{ eV} \cdot \text{s}
\end{align*}
\]

Permeability constant  \( \mu_o = 4\pi \times 10^{-7} \text{ N} / \text{A}^2 \)

Permittivity constant
\[
\begin{align*}
    \varepsilon_o &= 8.898 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \\
    \frac{1}{4\pi\varepsilon_o} &= 8.988 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2
\end{align*}
\]

Fine structure constant
\[
\alpha = \frac{e^2}{4\pi\varepsilon_o \hbar c} = 7.30 \times 10^{-3} = \frac{1}{137}
\]

Gravitational constant  \( G = 6.67 \times 10^{-11} \text{ m}^3 / \text{s}^2 \cdot \text{kg} \)

Avogadro’s number  \( N_A = 6.023 \times 10^{23} \text{ mole}^{-1} \)

Boltzmann’s constant  \( k = 1.381 \times 10^{-23} \text{ J/K} \\
= 8.617 \times 10^{-16} \text{ eV/K} \)

\( kT \) at room temperature  \( k \cdot 300 \text{ K} = 0.0258 \text{ eV} \)

Universal gas constant  \( R = 8.314 \text{ J/mole-K} \)

Stefan-Boltzmann constant  \( \sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 \)

Electron charge magnitude  \( e = 1.602 \times 10^{-19} \text{ C} \)

Electron rest mass  \( m_e = 9.109 \times 10^{-31} \text{ kg} \\
= 0.5110 \text{ MeV} / c^2 \)

Neutron rest mass  \( m_n = 1.675 \times 10^{-27} \text{ kg} \\
= 939.6 \text{ MeV} / c^2 \)

Proton rest mass  \( m_p = 1.672 \times 10^{-27} \text{ kg} \\
= 938.3 \text{ MeV} / c^2 \)

Deuteron rest mass  \( m_d = 3.343 \times 10^{-27} \text{ kg} \\
= 1875.6 \text{ MeV} / c^2 \)

Atomic mass unit (C\(^{12} = 12\))  \( u = 1.661 \times 10^{-27} \text{ kg} \\
= 931.5 \text{ MeV} / c^2 \)

Mass of earth  \( M_E = 5.98 \times 10^{24} \text{ kg} \)

Radius of earth  \( R_E = 6.37 \times 10^6 \text{ m} \)

Mass of sun  \( M_S = 1.99 \times 10^{30} \text{ kg} \)

Radius of sun  \( R_S = 6.96 \times 10^8 \text{ m} \)

Gravitational acceleration at earth’s surface  \( g = 9.81 \text{ m/s}^2 \)

Atmospheric pressure  \( = 1.01 \times 10^5 \text{ N/m}^2 \)

Radius of earth’s orbit  \( = 1.50 \times 10^{11} \text{ m} \)

Radius of moon’s orbit  \( = 3.84 \times 10^8 \text{ m} \)

Conversion Factors

\[
\begin{align*}
1 \text{ eV} &= 1.602 \times 10^{-19} \text{ J} \\
1 \text{ Å} &= 10^{-10} \text{ m} \\
1 \text{ barn (b)} &= 10^{-28} \text{ m}^2 \\
0^\circ \text{ Celsius} &= 273.16 \text{ K}
\end{align*}
\]

\[
\begin{align*}
1 \text{ J} &= 6.242 \times 10^{18} \text{ eV} \\
1 \text{ Fermi} &= 10^{-15} \text{ m} \\
1 \text{ in} &= 2.54 \text{ cm} \\
1 \text{ cal} &= 4.19 \text{ J}
\end{align*}
\]
**Problem 1:** Superman and the Flash decide to race from Seattle to New York (exactly 4600 km on a map) to see who is faster. Using his super vision, when Superman sees the clock in New York strike 8:00 PM they start off. Superman travels $0.75c$, and the Flash travels at $0.8c$.

(a) What time (to milliseconds) does the clock read when the Flash and Superman each arrive?

(b) How much farther does Superman believe he traveled than the Flash?

**Problem 2:** Using the mass of the electron $m_e = 0.511 \text{ MeV}/c^2$, the fine structure constant $\alpha = 1/137$, and $\hbar c = 197 \text{ MeV} \cdot \text{fm}$, give BOTH a symbolic and a numerical solution for the following:

(a) The Compton wavelength $\lambda_C$ of the electron.

(b) The Bohr radius $R_B$ of a Hydrogen atom.

(c) The speed $v$ of an electron in the lowest Bohr orbit.

**Problem 3:** Solve Laplace’s equation in the region shown in the figure, i.e. for $0 \leq x \leq a$, $0 \leq y \leq \infty$. The boundary conditions are $V = 0$ at $x = 0$ and $x = a$, and $V = V_0$ along $y = 0$. 

**Problem 4:** A 1.0 kg copper rod rests on two horizontal rails 1.0 m apart and carries a current of 50 A from one rail to the other. The coefficient of static friction between rod and rails is 0.60. What is the smallest magnetic field that would cause the rod to slide?
**Problem 5:** Show that in a lightly damped \( RLC \) circuit \( (R \ll \omega L) \), the fraction of the energy lost per oscillation is given approximately by \( 2\pi R/\omega L \).

**Problem 6:** Consider a two-dimensional rigid rotator in quantum mechanics that describes a molecule rotating in the \( xy \) plane. The Hamiltonian for this system is given by

\[ H_0 = L^2/2I, \]

where \( I \) is the moment of inertia of the rotator, and \( L = L_z \) is the orbital angular momentum along the \( z \)-axis.

(a) Solve for the eigenfunctions and eigenvalues of \( H_0 \).
(b) Suppose a weak perturbation is added to \( H_0 \), given by

\[ \lambda H' = 2\lambda \cos 2\phi, \]

where \( \phi \) is the angle in the \( xy \) plane that the rotator makes with the \( x \)-axis \( (0 \leq \phi \leq 2\pi) \). Calculate the effect of the perturbation on the 1st excited state of the rotator.

**Problem 7:** For a particle in an infinite square well of width \( a \):

\[ V(x) = \begin{cases} 0 & 0 \leq a \\ \infty & \text{otherwise} \end{cases} \]

calculate the first order correction to the energies and the wavefunctions due to a perturbing potential

\[ W(x) = a\omega_0 \delta(x - a/2), \]

where \( \omega_0 \) is a real constant with dimension of energy.

**Problem 8:** An electron can be found in a state \( \Psi_A \) near an isolated nucleus \( A \) with energy \( E_A \) or in a state \( \Psi_B \) near an isolated nucleus \( B \) with energy \( E_B \). The nuclei are now moved close together so that there is a non-vanishing Hamiltonian matrix element \( V = \langle \Psi_B | V | \Psi_A \rangle \) between the two states. Ignoring the direct overlap between the states \( (\langle \Psi_B | \Psi_A \rangle \approx 0) \), find the new energy levels of the one-electron, two nucleus system. Explain how your result might account for the formation of a stable diatomic molecule.