General Instructions

1. Each problem is to be done on a separate booklet. Label the front of each book with the identifying code letter you picked, the part number of the exam, and the number of the problem only; for example: A-I.6. Do not write your name or IIT ID number on any material handed in for grading.

2. Any numerical data not specified in a problem should be found in the table of constants at the front of the exam.

3. DON'T PANIC: It is not expected that each student will completely solve every problem. However, it is advisable to do a thorough job on those problems that you do solve.
Physical Constants

Speed of light in vacuum  \( c = 2.998 \times 10^8 \text{ m/s} \)

Planck’s constant  \( h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s} \)

\[
\frac{h}{2\pi} = 1.055 \times 10^{-34} \text{ J} \cdot \text{s}
\]

Permeability constant  \( \mu_0 = 4\pi \times 10^{-7} \text{ N/A}^2 \)

Permittivity constant  \( \frac{1}{4\pi\varepsilon_0} = 8.898 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \)

Fine structure constant  \( \alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c} = 7.30 \times 10^{-3} \approx \frac{1}{137} \)

Gravitational constant  \( G = 6.67 \times 10^{-11} \text{ m}^3 / \text{s}^2 / \text{kg} \)

Avogadro’s number  \( N_A = 6.023 \times 10^{23} \text{ mole}^{-1} \)

Boltzmann’s constant  \( k = 1.381 \times 10^{-23} \text{ J} / \text{K} \)

\( = 8.617 \times 10^{-5} \text{ eV} / \text{K} \)

\( kT \) at room temperature  \( k \cdot 300 \text{ K} = 0.0258 \text{ eV} \)

Universal gas constant  \( R = 8.314 \text{ J/mole-K} \)

Stefan-Boltzmann constant  \( \sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 \)

Electron charge magnitude  \( e = 1.602 \times 10^{-19} \text{ C} \)

Electron rest mass  \( m_e = 9.109 \times 10^{-31} \text{ kg} \)

\( = 0.5110 \text{ MeV}/c^2 \)

Neutron rest mass  \( m_n = 1.675 \times 10^{-27} \text{ kg} \)

\( = 939.6 \text{ MeV}/c^2 \)

Proton rest mass  \( m_p = 1.672 \times 10^{-27} \text{ kg} \)

\( = 938.3 \text{ MeV}/c^2 \)

Deuteron rest mass  \( m_d = 3.343 \times 10^{-27} \text{ kg} \)

\( = 1875.6 \text{ MeV}/c^2 \)

Atomic mass unit (\( C^{12} = 12 \))  \( u = 1.661 \times 10^{-27} \text{ kg} \)

\( = 931.5 \text{ MeV}/c^2 \)

Mass of earth  \( M_E = 5.98 \times 10^{24} \text{ kg} \)

Radius of earth  \( R_E = 6.37 \times 10^6 \text{ m} \)

Mass of sun  \( M_S = 1.99 \times 10^{30} \text{ kg} \)

Radius of sun  \( R_S = 6.96 \times 10^8 \text{ m} \)

Gravitational acceleration at earth’s surface  \( g = 9.81 \text{ m/s}^2 \)

Atmospheric pressure  \( = 1.01 \times 10^5 \text{ N/m}^2 \)

Radius of earth’s orbit  \( = 1.50 \times 10^{11} \text{ m} \)

Radius of moon’s orbit  \( = 3.84 \times 10^8 \text{ m} \)

Conversion Factors

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\begin{align*}
1 \text{ eV} &= 1.602 \times 10^{-19} \text{ J} \\
1 \text{ Å} &= 10^{-10} \text{ m} \\
1 \text{ barn (b)} &= 10^{-28} \text{ m}^2 \\
0^\circ \text{ Celsius} &= 273.16 \text{ K} \\
1 \text{ J} &= 6.242 \times 10^{18} \text{ eV} \\
1 \text{ Fermi} &= 10^{-15} \text{ m} \\
1 \text{ in} &= 2.54 \text{ cm} \\
1 \text{ cal} &= 4.19 \text{ J}
\end{align*}
\]
Problem 1: Two kickballs of radius $R$ and mass $m$ are placed in an upside down cylindrical bucket of diameter $3R$, with a third kickball on top (see figure). The ball on top is then removed.

(a) Describe the motion of the bucket in words.
(b) What is the minimum mass of the bucket necessary for the bucket to remain still?

Problem 2: A small ring slides on a wire which rotates uniformly at angular frequency $\omega$ in a force-free space. If the wire is straight and perpendicular to the axis of rotation, and the ring starts at the axis of rotation, what is the distance of the ring from the axis as a function of time?
Problem 3:
A board of length $L$ and mass $M$ can slide frictionlessly along a horizontal surface. A small block of mass $m$ initially rests on the board at its right end, as shown in the figure. The coefficient of friction between the block and the board is $\mu$. Starting from rest, the board is set in motion to the right with initial speed $v_0$. What is the smallest value of $v_0$ such that the block ends up sliding off the left end of the board? Assume the small block is sufficiently narrow relative to $L$ that its width can be neglected.

![Diagram of board and block](image1)

Problem 4:
Three identical point-like masses of mass $m$ are moving on a circle (see the figure). They are connected by identical springs of spring constant $k$. Determine the eigenfrequencies and fundamental (normal) modes of small oscillations of the system.

![Diagram of masses and springs](image2)

Problem 5:
A parallel beam of white light falls on a thin film of the refractive index $n = 1.33$. The incident angle $\theta_1 = 52^\circ$. What should the film thickness be equal to in order to maximize the yellow ($\lambda = 0.6 \mu m$) line intensity of the reflected beam?

![Diagram of light beam and film](image3)
Problem 6: A thermally insulated cylinder contains either Argon, Methane, or Air at room temperature. The contents are rapidly compressed to a volume 1/2 of the initial volume, and the pressure increases to approximately 2.5 of the initial pressure. Which gas is in the cylinder?

Problem 7:
A system in equilibrium at temperature $T$ of noninteracting spin-one particles of magnetic moment $\mu$ is placed in a constant magnetic field $B$. Derive an expression for the magnetization as a function of temperature.

Problem 8:
A potential energy of molecules in a certain central 3D field depends on a distance $r$ from the field center as $U = ar^2$, where $a$ is a positive constant. The gas temperature is $T$, the molecules concentration at the center of the field is $n_0$. Find:

a) The number of molecules at the distance from the center of the field between $r$ and $r + dr$.
b) The most probable distance between a molecule and the center of the field.
c) The fraction of molecules in a spherical layer between $r$ and $r + dr$.
d) How many times the molecules concentration at the center of the field will increase if the temperature increases $\eta$ times.